

HFS interval of the $2s$ state of hydrogen-like atoms and a constraint on a pseudovector boson with mass below $1 \text{ keV}/c^2$

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A constraint on a long-range spin-dependent interaction $\alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2) e^{-\lambda r}/r$, which can be induced by a pseudovector light boson, is presented. We study theoretical and experimental data on a specific difference $8 \times E_{\text{hfs}}(2s) - E_{\text{hfs}}(1s)$ for light two-body atoms. The spin-dependent coupling constant α'' of electron-nucleus interaction in hydrogen, deuterium and helium-3 ion is constrained at the level below a part in 10^{16} . The derived constraints are related to the range of masses below $4 \text{ keV}/c^2$.

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I. INTRODUCTION

A strong constraint from atomic physics can be set on a spin-dependent long-range interaction induced by a light axial-vector particle. In principle, a constraint on light particle with mass in keV/c^2 range may be derived by many means, involving cosmological estimation [1] and astrophysical phenomena [2]. Such constraints involve a number of parameters, such as the particle mass, its coupling to other particles, lifetime etc. In contrast to that a constraint based on limiting a possible deviation of electron-nucleus interaction in atomic distance range depends on only two parameters, namely the particle mass λ and a strength of interaction between an electron and a nucleus, mediated by the intermediate particle under consideration.

The previous constraint of this kind on spin-dependent interaction was derived from data on the hyperfine structure (HFS) interval of the $1s$ state in light hydrogen-like atoms [3, 4]. The result was for a particle substantially lighter than $4 \text{ keV}/c^2$ and the accuracy was limited either by the HFS experiment (muonium, positronium) or by an uncertainty of the related nuclear-effect contribution (hydrogen, deuterium). (It has been also extended there to heavier particles but with a reduced constraining strength.)

Here, to avoid uncertainties due to nuclear effects, we consider a specific difference of the $1s$ and $2s$ hyperfine intervals

$$D_{21} = 8 \times E_{\text{hfs}}(2s) - E_{\text{hfs}}(1s), \quad (1)$$

which is essentially free of such a problem [5, 6]. Experimental data with appropriate accuracy are available for hydrogen [9, 10], deuterium [11, 12] and helium-3 ion [13, 14] for their $1s$ and $2s$ hyperfine intervals. The corresponding data are summarized in Appendix A. For theoretical results, which are summarized in Appendix B, we follow [15].

Theory suggests that there is a massive cancelation of various contributions, which are proportional to the squared value of the wave function at origin

$$|\Psi_{ns}(0)|^2 \propto n^{-3}.$$

Those include various uncertain nuclear-effect terms, and a theoretical prediction for the difference has a very safe ground and has reached a high accuracy.

That is not the only theoretical advantage of the difference. The cancelation happens also with the leading term (see below) and because of that the fractional uncertainty of measurements of the difference is relatively low. Even with such a fractional accuracy the difference remains very sensitive to many higher-order effects.

Meantime, the theoretical accuracy in QED calculations for the HFS intervals is strongly affected by accuracy of our knowledge of fundamental constants required for the calculations and in particular of the nuclear magnetic moments (see, e.g. [6]). In the case the of difference the leading contributions have a large theoretical uncertainty, however, they cancel out in the difference and as result the theory of the difference is relatively immune to any problems in determination of the magnetic moments and other fundamental constants, which is indeed quite advantageous for theoretical calculations.

Returning to the leading term, the cancelation happens for the leading term to the ns HFS interval, a so-called Fermi contribution

$$\frac{E_F}{n^3} = C_s \frac{16\alpha}{3\pi n^3} \mu_B \mu_{\text{nuc}} R_\infty m_e^2, \quad (2)$$

where we apply relativistic units in which $\hbar = c = 1$, $e^2/(4\pi) = \alpha$ is the fine structure constant, m_e is the electron mass, R_∞ is the Rydberg constant, μ_B is the Bohr magneton and μ_{nuc} is the nuclear magnetic moment. The normalization constant C_s depends on the nuclear spin. In particular, $C_s = 1$ for the nuclear spin $1/2$ (hydrogen, helium-3 ion), while for the spin 1 (deuterium) an additional factor $C_s = 3/2$ appears.

A pseudovector particle, which interacts both with an electron and a nucleus, would induce a spin-dependent interaction (cf. contributions of the Z boson [7] and a_1

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meson [8] to the $1s$ HFS; see also [4]). If such an effect is present, the Coulomb exchange is modified by a spin-dependent term

$$-\frac{Z\alpha}{r} \rightarrow -\frac{Z[\alpha + \alpha''(\mathbf{s}_e \cdot \mathbf{s}_N)e^{-\lambda r}]}{r}, \quad (3)$$

where Z is the nuclear charge. Such a term is observable and may be used to produce a constraint on $\alpha''(\lambda)$ while comparing an actual value of D_{21} with theory.

In particular, in the limit

$$\lambda \ll Z\alpha m_e \sim Z \cdot 3.5 \text{ keV},$$

energy of each HFS interval is shifted by

$$\Delta E_{\text{hfs}}(ns) = -C_s \frac{2}{n^2} \frac{\alpha''}{\alpha} (Z^2 R_\infty) \quad (4)$$

and the related contribution to the difference is

$$\begin{aligned} \Delta D_{21} &= -2C_s \frac{\alpha''}{\alpha} (Z^2 R_\infty) \\ &= -0.9 \times 10^{18} \times C_s Z^2 \times \alpha'' \text{ Hz}, \end{aligned} \quad (5)$$

which should be compared with the difference between the related experimental and theoretical values. The factor $C_s Z^2$ is unity for hydrogen, $3/2$ for deuterium, and 4 for the helium-3 ion.

II. THE CONSTRAINT ON THE COUPLING CONSTANT OF A PSEUDOVECTOR BOSON

The present situation with experiments and theory of D_{21} is summarized in Table I, which covers all available data on determination of D_{21} in light two-body atoms. We also present there a value of α'' for an asymptotic region $\lambda \ll 1 \text{ keV}$. The result is indeed consistent with zero, since the theory and experiment are in perfect agreement.

| Atom | Experiment [kHz] | Theory [kHz] | α'' |
|-----------------|---------------------|-----------------|----------------------------------|
| H | 48.923(54) | 48.953(3) | $(3.3 \pm 5.9) \times 10^{-17}$ |
| D | 11.280(56) | 11.3125(5) | $(2.4 \pm 4.1) \times 10^{-17}$ |
| $^3\text{He}^+$ | -1189.979(71) | -1190.08(15) | $(-2.8 \pm 4.6) \times 10^{-17}$ |

TABLE I: Comparison of experiment and theory for the D_{21} value in light hydrogen-like atoms. A negative sign for the HFS difference for $^3\text{He}^+$ ion reflects the fact that the nuclear magnetic moment is negative, i.e., in contrast to other nuclei in the Table, its direction is antiparallel to the nuclear spin. The constraint on α'' is related to $\lambda \ll 1 \text{ keV}$. The confidence level of the constraint corresponds to one standard deviation.

If we consider α'' as a certain universal constant, an average value over the constraints in Table I is found as

$$\alpha''_{\text{av}} = (0.7 \pm 2.7) \times 10^{-17}. \quad (6)$$

To consider a constraint on a heavier intermediate particle, we have to calculate the contribution of the Yukawa correction in (3) to the D_{21} difference. As a result, the correction (5) should include an additional factor $\mathcal{F}_{12}(\lambda/(Z\alpha m_e))$ and the constraint takes the form

$$\alpha''(\lambda) = \frac{\alpha''_0}{\mathcal{F}_{12}(\lambda/(Z\alpha m_e))}, \quad (7)$$

where α''_0 is a constraint for $\lambda/(Z\alpha m_e) \ll 1$, listed in Table I and the profile function

$$\mathcal{F}_{12}(x) = 4 \left[\left(\frac{1}{1+x} \right)^2 - 2 \left(\frac{1}{1+x} \right)^3 + \frac{3}{2} \left(\frac{1}{1+x} \right)^4 \right] - \left(\frac{2}{2+x} \right)^2$$

satisfies the condition $\mathcal{F}_{12}(x \rightarrow 0) \rightarrow 1$.

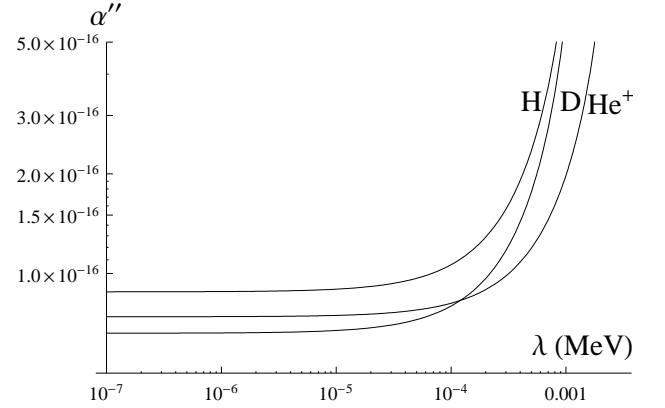


FIG. 1: Constraints on a pseudovector intermediate boson from D_{21} in hydrogen, deuterium and helium-3 ion. The lines present an upper bound for $|\alpha''|$. The confidence level corresponds to one standard deviation.

The related constraints extended to higher λ are presented in Fig. 1 [3], however it has sharp λ dependence and is not efficient above a few-keV level.

III. COMPARISON TO OTHER HFS CONSTRAINTS ON PSEUDOVECTOR BOSON

Because of low efficiency of the constraints in Eq. (7) above the keV region, we have to combine the results of this paper with constraint derived previously [4] from the data on the $1s$ HFS interval. Those constraints are weaker in the keV range but they are more suitable for extension to higher masses.

The overall constraint [3] from a study of the hyperfine intervals is summarized in Fig 2. Three low lines are from D_{21} (cf. Fig. 1) and the related constraints are much stronger in the one-keV region and below. However, the

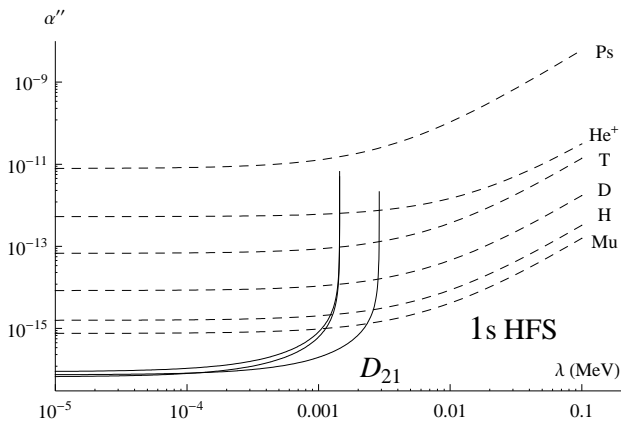


FIG. 2: Constraints on a pseudovector intermediate boson from the HFS study. The lines present the upper bound for $|\alpha''|$ from data on D_{21} (solid lines; see Fig. 1 for detail) and the $1s$ HFS interval (dashed lines) in various two-body atoms. The $1s$ results are from [4]. confidence level corresponds to one standard deviation.

lines related to the $1s$ HFS interval [4] produce stronger constraints for above a few keV.

That is expectable. In the case of the Yukawa radius, longer than atomic distances, the D_{21} constraints gain in accuracy because of the cancelation of the nuclear contributions which have large uncertainties. (The same mechanism turns the D_{21} difference into a powerful tool to test bound state QED [6].) However, once the radius is shorter than atomic distances, the Yukawa contribution becomes proportional to $|\Psi_{ns}(0)|^2$ and it is canceled out almost completely. Technically, that shows up as a special behavior of the function $\mathcal{F}_{12}(x) \propto x^{-4}$ at $x \rightarrow \infty$, while the related behavior for the $1s$ contribution [4]

$$\mathcal{F}_1(x) = \left(\frac{2}{2+x} \right)^2, \quad (8)$$

which in particular determines λ dependence of the $1s$ constraints in Fig. 2, is $\propto x^{-2}$. That makes the D_{21} difference insensitive to shorter-distance Yukawa spin-spin interactions.

For illustration, we present both profile functions in Fig. 3. Both are equal to unity for low λ and that is the area, where the constraints are the strongest. At large λ , both functions decrease to zero, which means that the Yukawa correction vanishes. However, as we mentioned, the behavior at high λ is different, which produces a different sensitivity for the high λ region. The results are obtained within a non-relativistic approximation. Taking into account relativistic effects does not change sharp-edge behavior of \mathcal{F}_{12} .

Thus, it is really fruitful to combine HFS constraints obtained by both methods: the D_{21} study for a longer wing of λ and the $1s$ HFS tests for the shorter one as summarized in Fig. 2. The constraints derived are complementary to various high-energy physics constraints re-

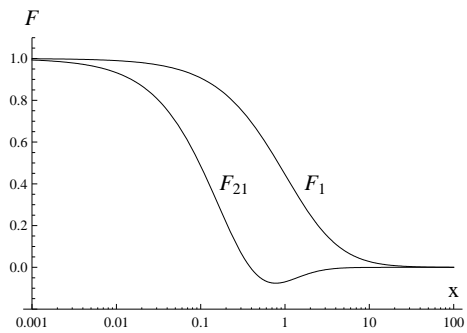


FIG. 3: The profile functions giving the upper bound for $|\alpha''|$ from data on D_{21} and the $1s$ HFS interval (in various two-body atoms. The $1s$ results are from [4].

viewed in [21].

To conclude, we remind that in particle physics the vertex for an interaction of a vector particle with a fermion is $-ig_V\gamma_\mu$, while for the pseudovector it is $-ig_A\gamma_5\gamma_\mu$. That means that the long-range interaction for particles x and y mediated by a pseudovector boson is of the form

$$\frac{\alpha_A(xy)(\boldsymbol{\sigma}_x \cdot \boldsymbol{\sigma}_y)}{r},$$

where $\alpha_A(xy) = g_A(x)g_A(y)/(4\pi)$. Comparing with substitute (3), where the spin-dependent coupling constant α'' is introduced, we note that $\alpha_A = \alpha''/4$ (since $\mathbf{s}_x = \boldsymbol{\sigma}_x/2$). That is rather the constant α_A that is the properly normalized coupling constant.

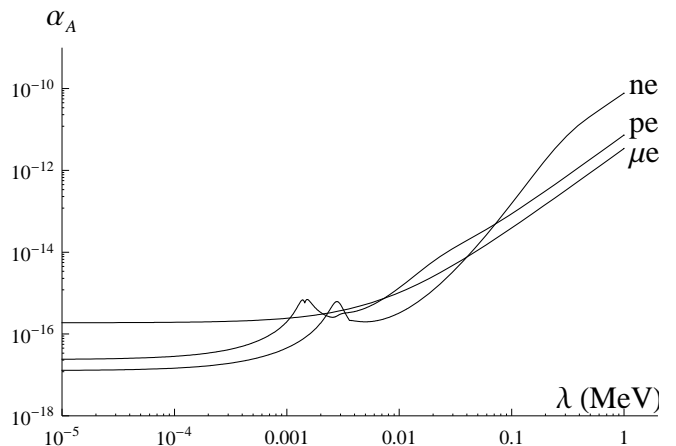


FIG. 4: Constraints on a pseudovector intermediate boson. The lines present the upper bound for the coupling constant $|\alpha_A(xy)|$ for $xy = pe, ne, \mu e$ from data on HFS intervals in various two-body atoms. The confidence level corresponds to one standard deviation.

We summarize in Fig. 4 the constraints on $\alpha_A(xe)$ for proton, neutron and muon (i.e. for $x = p, n, \mu$), where we have taken into account all results derived in [4] and in this paper. To separate proton and neutron contributions

we assume that nuclear binding effects can be neglected and thus for the deuteron we find

$$\alpha_A(de) = \frac{\alpha_A(pe) + \alpha_A(ne)}{2},$$

while the helion constant is assumed to be equal to a free neutron value ($\alpha_A(he) = \alpha_A(ne)$). Indeed, the binding effect could add a certain additional uncertainty, which is to be estimated. We do not think that would change the general situation.

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Appendix A: Summary on experimental data on the 1s and 2s HFS intervals in light two-body atoms

The experimental results on the metastable 2s state are available only for three hydrogen-like atoms, namely, for hydrogen, deuterium and helium-3 ion. Only a few measurements have been performed for over fifty years since fifties when the first results on the 2s HFS interval in hydrogen [16] and deuterium [17] atoms and helium-3 ion [18] were obtained. We summarize in Table II all obtained results.

| Atom | $E_{\text{HFS}}(\text{exp})$ [kHz] | Refs. |
|---------------------|---------------------------------------|-------|
| Hydrogen | 177 556.8343(67) | [10] |
| | 177 556.860(16) | [19] |
| | 177 556.785(29) | [20] |
| | 177 556.860(50) | [16] |
| Deuterium | 40 924.454(7) | [12] |
| | 40 924.439(20) | [17] |
| $^3\text{He}^+$ ion | -1083 354.980 7(88) | [14] |
| | -1083 354.99(20) | [18] |

TABLE II: All results on the 2s HFS interval in light hydrogen-like atoms obtained up to now. A negative sign for the $^3\text{He}^+$ ion reflects the fact that the nuclear magnetic moment is negative and thus its direction is antiparallel to the nuclear spin.

Since only these three atoms are important for calculations of a specific difference of the HFS intervals in the 1s and 2s states, we collect in Table III the experimental results on the 1s HFS interval for involved atoms.

The results on the difference D_{21} , based on the most accurate experimental results, are present in Table I of the paper.

| Atom | $E_{\text{HFS}}(\text{exp})$ [kHz] | Ref. |
|---------------------|---------------------------------------|------|
| Hydrogen | 1 420 405.751 768(1) | [9] |
| Deuterium | 327 384.352 522(2) | [11] |
| $^3\text{He}^+$ ion | - 8 665 649.867(10) | [13] |

TABLE III: The most accurate results for the 1s HFS interval in those light hydrogen-like atoms, for which the results on the 2s HFS interval are available.

Appendix B: Summary on theory of the D_{21} difference in light two-body atoms

A detailed review on theory of the D_{21} difference in hydrogen, deuterium and helium-3 ion can be found in [5, 6]. The results are summarized in Table IV. ‘QED3’ and ‘QED4’ stands for pure QED corrections in units of the Fermi energy E_F , defined in (2).

There are three small parameters in QED theory: α stands for QED loops and is for the QED perturbation effects, $Z\alpha$ is for the Coulomb strength and describes binding effects, while the mass ratio m/M (electron-to-nucleus) is for the recoil effects in two-body atoms. Theoretical evaluations have a certain history, being started in [22–24], shortly after the first results on the 2s HFS interval were achieved [16–18].

The QED3 term involves various combinations of these three parameters up to the third-order, which were mainly calculated long time ago. A more recent development was due to the fourth-order contributions (QED4), which include the fourth-order contributions and, due to higher-order nuclear effects.

| Contribution to D_{21} | Hydrogen [kHz] | Deuterium [kHz] | $^3\text{He}^+$ ion [kHz] |
|-----------------------------|-------------------|--------------------|------------------------------|
| $D_{21}(\text{QED3})$ | 48.937 | 11.305 6 | -1 189.253 |
| $D_{21}(\text{QED4})$ | 0.018(5) | 0.004 4(10) | -1.13(14) |
| $D_{21}(\text{Nucl})$ | -0.002 | 0.002 6(2) | 0.307(35) |
| $D_{21}(\text{total})$ | 48.953(5) | 11.312 5(10) | -1 190.08(15) |

TABLE IV: Theory of the specific difference D_{21} in light hydrogen-like atoms [15]. The numerical results are presented for the related frequency D_{21}/h . QED3 and QED4 stands for the third- and fourth-order QED corrections in units of the Fermi energy E_F (see [5, 6] for detail).

As we mentioned above, there is a substantial cancellation of the nuclear-structure contribution in difference D_{21} . The leading term, which takes into account nuclear charge and magnetic moment distribution, cancels completely. However, certain higher-order nuclear-effect contributions survive the cancelations and they are denoted as ‘Nucl’. Those higher-order terms were found in [15].

For QED3 terms and for higher-order nuclear effects we follow [6], while for the QED4 terms we apply results

of [15], a recent correction in which follows a reexamination of the former QED4 calculation in [5] and numerical

medium- Z calculation of one-loop effects in [25] (cf. [26]).

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